Total No. of Questions : 8] *[Total No. of Printed Pages : 3*

Roll No .....

BE-3001 (EE/EX)-CBGS

B.E., III Semester

Examination, December 2020

Choice Based Grading System (CBGS)

Mathematics - III

Time : Three Hours

Maximum Marks : 70

Note: i) Attempt any five questions.

ii) All questions carry equal marks.

1. a) Express the function $f(x) = x \sin x$, as a Fourier series in $-\pi \le x \le \pi$. Hence deduce that

$$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ldots=\frac{\pi-2}{4}$$ 7

b) Find the Fourier sine transform of $f(x) = e^{ax}/x$. 5

c) State and prove Linearity property of Fourier transform. 2

2. a) Solve the integral equation

$$\int_0^\infty f(x)\cos sx\,dx=\begin{cases}1-s, & 0 \le s \le 1\\ 0, & s>1\end{cases}$$

Hence prove that $\int_0^\infty \frac{\sin^2 t}{t^2}dt=\frac{\pi}{2}$. 7

b) Find the Laplace transform of $\frac{1-\cos t}{t^2}$ 7

3. a) Find the inverse Laplace transform of $\frac{1}{S^4+4}$ 7

b) If $u-v = (x-y)(x^2+4xy+y^2)$ and $f(z) = u+iv$ is an analytic function of z. Find $f(z)$. 7

4. a) Evaluate $\int_c \frac{z^2+1}{z^2-1}dz$ where C is 7

i) $|z| = 3/2$

ii) $|z| = 1/2$

iii) $|z-1| = 1$

b) If the directional derivative of $\phi = ax^2y + by^2z + cz^2x$ at the point (1,1,1) has maximum magnitude 15 in the direction of $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$, find a, b and c. 7

5. a) Evaluate surface integral $\iint \vec{F}.\hat{n}\, ds$, where $\vec{F} = \left(x^2+y^2+z^2\right)\left(\hat{i}+\hat{j}+\hat{k}\right)$, S is the surface of the tetrahedron $x = 0$, $y = 0$, $z = 0$, $x + y + z = 2$ and $\hat{n}$ is the unit normal in the outward direction to the closed surface S. 7

b) Find the Fourier series for $f(x)=\begin{cases}-\pi & -\pi<x<0\\ x & 0<x<\pi\end{cases}$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\quad=\frac{\pi^2}{8}$ 7

6. a) A periodic function of period 4 is defined as $f(x)=|x|, -2<x<2$. Find its Fourier series expansion. 7

b) Find the Fourier cosine transform of $\frac{1}{1+x^2}$. 7

7. a) Find the Fourier sine transform of $\frac{1}{x}$. 3

b) State and prove modulation property of Fourier transform. 4

c) Solve $y''+2y'+5y=e^{-x}\sin x$ where y(0)= 0, y'(0)=1by Laplace transform method. 7

8. a) Use convolution theorem to find $L^{-1}\left[\frac{1}{(s^2+a^2)^2}\right]$ 7

b) Evaluate $\int_{-\infty}^{\infty}\frac{x\sin\pi x}{x^2+2x+5}dx$ by contour integration. 7

******